NEW EXPONENTIAL CUMULATIVE HAZARD METHOD FOR GENERATING CONTINUOUS FAMILY DISTRIBUTIONS

Salma Omar Bleed
Statistics Department, Alasmarya Islamic University, Zliten, Libya

S.Ali@asmary.edu.ly & Salmableed@yahoo.com

Abstract
The article aims to expand the use of the cumulative hazard function and cumulative generalized exponential distribution of Gupta and Kunda (1999) for introducing a new method for generating families from continuous distributions called the Exponential Cumulative Hazard (ECH) method. It's providing some of well–known methods and distributions embedded within the proposed method. New 5–parameter uniform distributions with 3–shape parameters and bathtub hazard function are introduced as a practical example to support the proposed method. Finally, application on real data–set is provided.

Keywords: Exponential cumulative hazard method, modified Weibull, Kumaraswamy distributions; Kumaraswamy inverse exponential
1. Introduction

There is an interest in presenting a new generalized method for some of the previously presented methods and distributions. The purpose of the new proposed method in this article is to expand the use of the cumulative hazard function and the cumulative generalized exponential distribution of (Gupta and Kunda (1999)). It's called the Exponential Cumulative Hazard (ECH) method. Kupta and Kunda relied on the Gompertz-Verhulst distribution function to find a generalized distribution of the exponential distribution. Depending on Kupta and Kunda's method and the cumulative hazard function, the new method will be described in details as follows: the cumulative generalized exponential distribution of (Gupta and Kunda (1999)) given by

\[ F(t) = (1 - e^{-\lambda t})^\theta \quad ; \quad t > 0 \quad \lambda, \theta > 0 \]  

(1)

with cumulative hazard function

\[ H(t) = -\ln \left[ 1 - F^\alpha(t) \right] \quad ; \quad t > 0 \quad \alpha > 0 \]  

(2)

Replacing the random variable \( t \) in (1) by the cumulative hazard function (2), the cumulative distribution function (cdf) of the new proposed method will be expressed as

\[ Q(t) = \left[ 1 - \left( 1 - F^\alpha(t) \right)^\lambda \right]^{\theta} \quad ; \quad t > 0 \quad \alpha, \lambda, \theta > 0 \]

with pdf

\[ q(t) = \alpha \lambda \theta f(t) F^{\alpha-1}(t) \left[ 1 - F^\alpha(t) \right]^\lambda \left[ 1 - \left( 1 - F^\alpha(t) \right)^\lambda \right]^{\theta-1} \quad ; \quad t > 0 \quad \alpha, \lambda, \theta > 0 \]

Note that the proposed method to generate various statistical distributions provides more options for analyzing data restricted to the time interval, while the \( F(t); t > 0 \) with any arbitrary distribution \((0, \infty)\) exponentiated Kumaraswamy distribution provides for analyzing data restricted to the interval \((0, 1)\) with cdf \( F(t) = t; 0 < t < 1 \).
The rest of this article is organized as follows: the generalization of generalized some of well-known distributions with sub-methods are presented in Section 2. Generalization of generalized some of well-known distributions with sub-models are presented in Section 3. New 5-parameter uniform distributions with some mathematical properties are provided in Section 4. Real data is used to illustrate the usefulness of the 5-parameter uniform distributions for modeling lifetime data is provided in Section 5.

### 2. Generalization of Generalized Some Distributions with Sub-Methods

#### 2.1. Generalization of generalized exponential distributions

<table>
<thead>
<tr>
<th>New Generalization</th>
<th>Base cdf</th>
<th>New cdf</th>
</tr>
</thead>
<tbody>
<tr>
<td>generalized</td>
<td>$F(t), t &gt; 0$</td>
<td>$Q(t) = \left[F^\alpha(t)\right]^\theta$ with $\lambda = 1$</td>
</tr>
</tbody>
</table>

Well-known Sub Parameters cdf

- generalized exponential distributions
  - $\alpha = \lambda = 1$
  - $Q(t) = [F(t)]^\theta$

of (Gupta and Kunda (1999))

#### 2.2. Generalization of exponential generalized distributions

<table>
<thead>
<tr>
<th>New Generalization</th>
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</thead>
<tbody>
<tr>
<td>generalized</td>
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<td>$Q(t) = \left[1 - \left(1 - F^\alpha(t)\right)^\lambda\right]^\theta$</td>
</tr>
</tbody>
</table>

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108
Well-known Sub Method Parameters cdf
exponential generalized distributions of \( \alpha = 1 \)
\( Q(t) = \left( 1 - (1 - F(t))^2 \right)^\theta \)

2.3. Generalization of type-II Topp–Leone-G family of distributions

New Generalization Base cdf New cdf
generalized type-II Topp–Leone-G family of distributions
\( F(t), t > 0 \)
\( Q(t) = \left( 1 - \left( 1 - F(t)^{2} \right)^{\alpha} \right)^{\theta} \) with
\( \alpha = 2 \)

Well-known Sub Method Parameters cdf
type-II Topp–Leone-G family of distributions \( \theta = 1 \)
\( Q(t) = 1 - \left( 1 - F(t)^{2} \right)^{\alpha} \)

2.4. Generalization of generalized Kumaraswamy-G family of distributions

New Generalization Base cdf New cdf
Generalized generalized Kumaraswamy-G family of distributions
\( F(t) = \frac{1 - \left( 1 - \beta F(t)^{a} \right)^{b}}{1 - (1 - \beta)^{b}} \)
\( Q(t) = 1 - \left( 1 - \left( \frac{1 - (1 - \beta F(t)^{a})^{b}}{1 - (1 - \beta)^{b}} \right)^{\alpha} \right)^{\theta} \)
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Well-known Sub Method Parameters cdf

generalized Kumaraswamy-G family of distributions of (Zohdy et al. (2019))

\[ \alpha = \lambda = \theta = 1 \]
\[ F(t) = \frac{1 - \left[ (1 - \beta) F(t)^a \right]^b}{1 - (1 - \beta)^b} \]

3. Generalization of Generalized some Distributions with Sub-Models

3.1

New Generalization Base cdf New cdf

generalized generalized inverse generalized Weibull distribution (generalized generalization of Kumaraswamy inverse exponential distribution)

\[ F(t) = e^{-\left( \frac{a}{t} \right)^b} \]
\[ Q(t) = \left[ 1 - e^{-\alpha \left( \frac{a}{t} \right)^b} \right]^{-\theta} \]

Well-known Sub Models Parameters cdf

generalized inverse generalized Weibull distribution of \( \theta = 1 \) (Kanchan, Neetu, and Suresh (2014))

\[ Q(t) = 1 - e^{-\alpha \left( \frac{a}{t} \right)^b} \]

inverse generalized Weibull distribution of
(Kanchan, Neetu, and Suresh (2014))

Kumaraswamy inverse exponential distribution of
(Oguntunde, Babatunde and Ogunmola (2014))

generalized inverted exponential distribution of
(Abuammoh and Alshingiti (2009))

inverse Weibull distribution of
(Drapella (1993))

inverted exponential distribution of
(Keller and Kamath (1982))

New Generalization  Base cdf  New cdf

generalized generalized exponentiated Weibull

\[ F(t) = 1 - e^{-(at)^b} \]
\[ Q(t) = \left( 1 - e^{-(at)^b} \right)^{\lambda} \]
NEW EXPONENTIAL CUMULATIVE HAZARD METHOD FOR GENERATING CONTINUOUS FAMILY DISTRIBUTIONS

<table>
<thead>
<tr>
<th>Well-known Sub Model</th>
<th>Parameters</th>
<th>cdf</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-parameter exponential distributions of (Bukoye and Oyeyemi (2018))</td>
<td>$b = 1, a = \frac{1}{\gamma}$</td>
<td>$Q(t) = \left[ 1 - \left(1 - e^{-\gamma} \right)^{-\frac{a}{\gamma}} \right]^\theta$</td>
</tr>
<tr>
<td>generalized exponentiated distribution of (Ramesh, Pushpa, and Rameshwar (1998))</td>
<td>$\lambda = \phi = b = 1$</td>
<td>$Q(t) = \left(1 - e^{-\phi t} \right)^{\alpha}$</td>
</tr>
<tr>
<td>generalized Weibull distribution of (Mudholkar and Srivastava (1993))</td>
<td>$\alpha = \lambda = 1$</td>
<td>$Q(t) = \left(1 - e^{-\lambda t} \right)^{\theta}$</td>
</tr>
<tr>
<td>exponential distribution of (Bain (1974))</td>
<td>$\theta = \phi = \alpha = b = 1$</td>
<td>$Q(t) = 1 - e^{-\alpha t}$</td>
</tr>
</tbody>
</table>

New Generalization

<table>
<thead>
<tr>
<th>Base cdf</th>
<th>New cdf</th>
</tr>
</thead>
<tbody>
<tr>
<td>generalized generalized Rayleigh distribution and generalized Kumaraswamy Weibull distribution</td>
<td>$F(t) = 1 - e^{-\alpha t b}$</td>
</tr>
</tbody>
</table>

Well-known Sub Models

<table>
<thead>
<tr>
<th>Parameters</th>
<th>cdf</th>
</tr>
</thead>
</table>
Kumaraswamy Weibull distribution of (Cordeiro, Ortega, and Nadarajah (2010))

\[ \theta = 1 \]

\[ Q(t) = 1 - \left( 1 - e^{-a t^b} \right)^\alpha \]

generalized Rayleigh distribution of (Kundu and Raqab (2005))

\[ \alpha = \lambda = 1, b = 2 \]

\[ Q(t) = \left( 1 - e^{-a t^2} \right)^\theta \]

Exponentiated Weibull distribution of (Mudholkar and Srivastava (1993))

\[ \alpha = \lambda = 1 \]

\[ Q(t) = \left( 1 - e^{-a t^b} \right)^\theta \]

Rayleigh distribution of (Bain (1974))

\[ \alpha = \lambda = \theta = 1, b = 2 \]

\[ Q(t) = 1 - e^{-a t^2} \]

Weibull distribution of (Weibull (1951))

\[ \alpha = \lambda = \theta = 1 \]

3.4. Generalization of inverted exponential distribution (exponentiated inverse Weibull)

New Generalization

Base cdf

New cdf

generalized inverted exponential distribution

(generalized Exponentiated Inverse Weibull))

\[ F(t) = 1 - e^{-(at)^b} \]

\[ Q(t) = \left[ 1 - \left( 1 - e^{-(at)^b} \right)^\alpha \right]^{\lambda^\theta} \]

Well-known Sub Models

Parameters

cdf

inverted exponential distribution of (Lin, Duran, and Lewis (1989))

\[ \alpha = \lambda = \theta = b = 1 \]

\[ Q(t) = 1 - e^{-(at)^{-1}} \]
### 3.5. Generalization of exponentiated generalized modified Weibull distribution

<table>
<thead>
<tr>
<th>New Generalization</th>
<th>Base cdf</th>
<th>New cdf</th>
</tr>
</thead>
<tbody>
<tr>
<td>generalized exponentiated generalized modified Weibull distribution</td>
<td>( F(t) = 1 - e^{-(\beta t + \mu t^\gamma)} )</td>
<td>( Q(t) = 1 - \left[ 1 - e^{-(\beta t + \mu t^\gamma)} \right]^\alpha \theta )</td>
</tr>
</tbody>
</table>

- \( \gamma > 0, \beta, \mu \geq 0 \) and \( \beta + \mu > 0 \)

**Well-known Sub Models**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>cdf</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-parameter exponentiated generalized modified Weibull distribution of (Gokarna and Ibrahim (2015))</td>
<td>( \alpha = 1 )</td>
</tr>
<tr>
<td>4-parameter exponentiated modified Weibull distribution of (Elbatal (2011))</td>
<td>( \alpha = \lambda = 1 )</td>
</tr>
</tbody>
</table>

### New Generalization

<table>
<thead>
<tr>
<th>Base cdf</th>
<th>New cdf</th>
</tr>
</thead>
<tbody>
<tr>
<td>generalized inverted Kumaraswamy distribution</td>
<td>( F(t) = \frac{t}{1+t} )</td>
</tr>
</tbody>
</table>
Well-known Sub Models

Parameters cdf

inverted Kumaraswamy distribution of (Abd Al-Fattah, Al-Helbawy, and Al-Dayian (2017))

$$Q(t) = \left[1 - \left(1 + t\right)^{-\lambda}\right]^{-\theta}$$

4. Five Parameter Uniform Distributions

Let $$Q(t)$$ be the cdf of the proposed method and $$F(t) = \frac{t-a}{b-a}$$, $$a \leq t \leq b$$ is the cdf of the uniform distribution with parameters $$a,b > 0$$. Then the cdf $$Q(t)$$ of the 5-parameter uniform distributions with 3-shape parameters $$(\alpha, \lambda, \theta)$$, scale parameter $$(b)$$ and location parameter $$(a)$$, respectively is given by

$$Q(t) = \left(1 - \xi^\lambda\right)^\theta$$

with pdf

$$q(t) = \frac{\theta \lambda a}{(b-a)} T^{\alpha-1} \xi^\lambda - 1 \left(1 - \xi^\lambda\right)^{\theta-1}$$

(3),

reliability function

$$R(t) = 1 - \left(1 - \xi^\lambda\right)^\theta$$

and hazard function

$$h(t) = \frac{\theta \lambda a}{(b-a)} \left[\xi^{\lambda - 1} \left(1 - \xi^\lambda\right)^{-\theta - 1}\right]^{-1}$$

where

$$T = \frac{t-a}{b-a}$$, $$\xi = 1 - T^{\alpha}$$, $$a \leq t \leq b$$, $$\lambda, \theta, \alpha, a, b > 0$$. 

Figure 1, illustrate plot the pdf of the 5-parameter uniform distributions under different values of the 3-shape parameters \((\alpha, \lambda, \theta)\). It is noted that the pdf takes many shapes, and this gives a good advantage to the distribution in that it is a more flexible and more suitable distribution for many life phenomena. The plots of the cdf and the reliability indicate increasing cdf with decreasing reliability function, see Figure 2 and Figure 3, respectively.

The bathtub curve of three life stages can be modeled with the 5-parameter uniform distributions and varying values of \(\lambda > 1, \alpha, \theta < 1\). In addition, the failure rate of units exhibiting increases with time with 5-parameter uniform and varying values of \(\alpha, \lambda, \theta > 1\) and decreases with varying values of \(\alpha, \theta < 1, \lambda > 7\).
The moment generating function $m_t(g)$ of a r.v $t$ distributed as 5-parameter uniform distributions is

$$m_t(g) = \frac{\theta e^{ag} \sum_{n=0}^{\infty} (-1)^n \left(\frac{\theta - 1}{n}\right) m!}{\sum_{m=0}^{\infty} \frac{g^m(b-a)^m}{m!} \beta(\eta(n+1))}$$

with $r$th row moments

$$\mu'_r = \frac{\theta \sum_{n=0}^{\infty} (-1)^n \left(\frac{\theta - 1}{n}\right)}{\sum_{m=0}^{\infty} \frac{g^m(b-a)^m}{m!} \beta(\eta(n+1))}$$

and quantile function

$$\text{Quantile} = a + (b-a) \left[1 - \left(1 - p^{\theta-1}\right)^{\lambda-1}\right]^{-1}.$$

For a random sample $t = t_1, t_2, ..., t_n$ with size $n$ from 5-parameter uniform distributions, let the order statistics be $t_{(1)} < t_{(2)} < ... < t_{(n)}$. Utilizing expression (3), the likelihood with log likelihood function for $t$ are by definition

$$L = \frac{(\theta \alpha)^n}{(b-a)^n} \prod_{i=1}^{n} (T_i^{\alpha-1} \xi_i^{\lambda-1}) \ln(1 - \xi_i^\lambda),$$

where $T_i = \frac{t_i - a}{b-a}$, $\xi_i = l - T_i^\alpha$, $t_{(1)} = \hat{a}$, $t_{(n)} = \hat{b}$. By taking the partial derivative of $\ln L$ with respect to $\theta$, the closed form of $\hat{\theta}$ is gotten as

$$\hat{\theta} = n \left[\sum_{i=1}^{n} \ln(1 - \xi_i^\lambda)\right]^{-1}.$$

Unfortunately, to get $\hat{\alpha}$ and $\hat{\lambda}$, the computation of $\ln L$ cannot be performed by solving the following two normal equations for $\lambda, \alpha$. Therefore, they will be solving numerically.
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\[
\frac{\partial \ln L(t_i; \psi)}{\partial \lambda} = \frac{n}{\lambda} + \frac{n}{\lambda} \sum_{i=1}^{n} \left( 1 - \left( \theta - I \right) \left[ \xi_i - \lambda - I \right]^{-1} \right) \ln \xi_i = 0
\]

\[
\frac{\partial \ln L(t_i; \psi)}{\partial \alpha} = \frac{n}{\alpha} + \frac{n}{\alpha} \sum_{i=1}^{n} \left( 1 + \left( \xi_i - \alpha - I \right)^{-1} \right) \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \left[ \xi_i - \lambda - I \right]^{-1} \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \ln T_i = 0
\]

The elements of the Fisher information matrix (FI) for the MLE can be obtained as the expectations of the negative of the 2nd partial derivatives, and the asymptotic variance-covariance matrix for the MLE is defined as the inverse of the Fisher information matrix. The 2nd partial derivatives with respect to \( \alpha, \lambda \) and \( \theta \) are

\[
\frac{\partial^2 \ln L(t_i; \psi)}{\partial \theta^2} = -\frac{n}{\theta^2}
\]

\[
\frac{\partial^2 \ln L(t_i; \psi)}{\partial \alpha^2} = -\frac{n}{\alpha^2} \left( 1 - \alpha^{\theta} - I \right) \left[ \xi_i - \alpha - I \right]^{-1} \left( I - \alpha^{\theta} - I \right) \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \left[ \xi_i - \lambda - I \right]^{-1} \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \ln T_i
\]

\[
\frac{\partial^2 \ln L(t_i; \psi)}{\partial \theta \partial \lambda} = -\frac{n}{\sum_{i=1}^{n} \left( \xi_i - \lambda - I \right)^{-1} \ln \xi_i
\]

\[
\frac{\partial^2 \ln L(t_i; \psi)}{\partial \theta \partial \alpha} = \lambda \sum_{i=1}^{n} \left( T_i - \alpha - I \right) \left[ \xi_i - \alpha - I \right]^{-1} \left( \xi_i - \lambda - I \right]^{-1} \ln T_i
\]

\[
\frac{\partial^2 \ln L(t_i; \psi)}{\partial \lambda \partial \alpha} = \frac{n}{\sum_{i=1}^{n} \left( \xi_i - \lambda - I \right) \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \left( \lambda (\theta - I) \left( 1 - T_i \alpha \right)^{-1} \right) \ln T_i
\]
5. Application

The data in the table below are 55 smiling times, in seconds, of an eight-week-old baby. The smiling times, in seconds, follow 5-parameter uniform distributions between 0.7 and 22.8 seconds.

<table>
<thead>
<tr>
<th></th>
<th>10.4</th>
<th>19.6</th>
<th>18.8</th>
<th>13.9</th>
<th>17.8</th>
<th>16.8</th>
<th>21.6</th>
<th>17.9</th>
<th>12.5</th>
<th>11.1</th>
<th>4.9</th>
<th>8.9</th>
<th>9.4</th>
<th>9.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.8</td>
<td>14.8</td>
<td>22.8</td>
<td>20.0</td>
<td>15.9</td>
<td>16.3</td>
<td>13.4</td>
<td>17.1</td>
<td>14.5</td>
<td>19.0</td>
<td>22.8</td>
<td>7.6</td>
<td>10.0</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td>0.7</td>
<td>8.9</td>
<td>11.9</td>
<td>10.9</td>
<td>7.3</td>
<td>5.9</td>
<td>3.7</td>
<td>17.9</td>
<td>19.2</td>
<td>9.8</td>
<td>6.7</td>
<td>7.8</td>
<td>11.6</td>
<td></td>
</tr>
<tr>
<td>5.8</td>
<td>6.9</td>
<td>2.6</td>
<td>5.8</td>
<td>21.7</td>
<td>11.8</td>
<td>3.4</td>
<td>2.1</td>
<td>4.5</td>
<td>6.3</td>
<td>10.7</td>
<td>13.8</td>
<td>18.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the data of this example, the distribution parameters with mean squared error (MSE) and 99% lower bound (L.C.I) and upper bound confidence interval (U.C.I) were estimated. As well as, mean, median, variance, coefficient of skewness, and coefficient of kurtosis of the data were calculated, see table 1 and table 2.

Table 1: MSE and 99% C.I for the Distribution Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Initial Values</th>
<th>MLE</th>
<th>MSE</th>
<th>L.C.I</th>
<th>U.C.I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.25</td>
<td>0.261</td>
<td>1.12510E-4</td>
<td>0.233</td>
<td>0.288</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.15</td>
<td>1.179</td>
<td>8.18810E-4</td>
<td>1.132</td>
<td>1.226</td>
</tr>
<tr>
<td>$\theta$</td>
<td>5.5</td>
<td>5.709</td>
<td>0.044</td>
<td>3.4</td>
<td>8.019</td>
</tr>
</tbody>
</table>

| $\alpha$   | 0.5            | 0.513| 1.67510E-4 | 0.421  | 0.605  |
| $\lambda$  | 1.5            | 1.189| 0.096    | 0.987  | 1.392  |
| $\theta$   | 2.75           | 2.703| 2.22510E-3| 1.309  | 4.096  |

| $\alpha$   | 0.35           | 0.382| 1.01710E-3| 0.325  | 0.439  |
| $\lambda$  | 1.25           | 1.185| 4.2510E-3 | 1.103  | 1.267  |
| $\theta$   | 3.5            | 3.742| 0.059    | 2.151  | 5.334  |

| $\alpha$   | 0.2            | 0.199| 1.14310E-6| 0.183  | 0.215  |
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\[ \lambda \begin{array}{cccc} 1.125 & 1.175 & 2.5181 \times 10^{-3} & 1.141 \\ \theta & 7.76 & 7.711 & 2.4111 \times 10^{-3} \\ \end{array} \]

Table 2: Mean, Median, Variance, Skewness, and Kurtosis of the data

<table>
<thead>
<tr>
<th>Initial Values ((\alpha, \lambda, \theta))</th>
<th>Mean</th>
<th>Median</th>
<th>Variance</th>
<th>Coefficient of Skewness</th>
<th>Coefficient of Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1.150</td>
<td>5.50</td>
<td>11.811</td>
<td>11.845</td>
<td>36.946</td>
</tr>
<tr>
<td>0.50</td>
<td>1.500</td>
<td>2.75</td>
<td>11.841</td>
<td>11.906</td>
<td>37.186</td>
</tr>
<tr>
<td>0.35</td>
<td>1.250</td>
<td>3.50</td>
<td>11.826</td>
<td>11.875</td>
<td>37.052</td>
</tr>
<tr>
<td>0.20</td>
<td>1.125</td>
<td>7.76</td>
<td>11.803</td>
<td>11.827</td>
<td>36.882</td>
</tr>
</tbody>
</table>

6. Conclusions

Depending on Kupta and Kunda's method and the cumulative hazard function, the new method called the Exponential Cumulative Hazard (ECH) method is introduced. It's providing some of well-known methods and distributions embedded within the proposed method. This method is considered one of the most important methods that enables us to generate methods of some of previously proposed methods and families from continuous distributions, such as generalization of inverted exponential distribution of (Lin, Duran, and Lewis (1989)), generalized Weibull distribution and exponentiated Weibull of (Mudholkar and Srivastava (1993)), generalized exponentiated distribution of (Ramesh, Pushpa, and Rameshwar (1998), generalized exponential distributions of (Gupta and Kunda (1999)), generalized Rayleigh distribution of (Kundu and Raqab (2005)), Kumaraswamy Weibull distribution of (Cordeiro, Ortega, and Nadarajah (2010), 4-parameter exponentiated modified Weibull distribution of (Elbatal (2011)), exponential generalized distributions of (Cordeiro, Ortega, and Daniel (2013)), inverse generalized Weibull
distribution and generalized inverse generalized Weibull distribution of (Kanchan, Neetu, and Suresh(2014)), Kumaraswamy inverse exponential distribution of (Oguntunde, Babatunde and Ogunmola (2014)), generalization of 5-parameter exponentiated generalized modified Weibull distribution of (Gokarna and Ibrahim (2015)), generalization of inverted Kumaraswamy distribution of (Abd Al-Fattah, El-Helbawy, and Al-Dayian (2017)), generalization of type-II Topp–Leone-G family of distributions of (Elgarhy et al. (2018)), 4-parameter exponential distributions of (Bukoye and Oyeyemi (2018)), and generalization of generalized Kumaraswamy-G family of distributions of (Zohdy et al. (2019)). Finally, 5-parameter uniform distributions with some mathematical properties with application on real data-set are provided. The new distribution takes many shapes with hazard function modeled as the bathtub curve of three life stages, and this gives a good advantage to the distribution in that it is a more flexible and more suitable distribution for many life phenomena.

References

NEW EXPONENTIAL CUMULATIVE HAZARD METHOD FOR GENERATING CONTINUOUS FAMILY DISTRIBUTIONS


