The Cubic Bezier-Ball Like curve with Shape Parameters

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Abstract

In this paper the basic spotlight is construct new cubic basis functions to generate rational cubic Bezier-Ball Like curve with two shape parameters. The current study the Bezier-Ball Like curve analogous to cubic Bezier curve, Ball curve, and Timmer curve at specific value of shape parameters. The shape of the curve can be modified by change the value of shape parameters and weights. In this work the Arabic font can be represented using Bezier-Ball Like curve.

Key words: Computer Aided Geometric Design, Cubic Bezier-Ball Like curve, Rational cubic Bezier-Ball Like curve, Shape parameters, Shape control.

1 Introduction

The curve design is the most important topic of CAGD (Computer Aided Geometric Design) and Computer Graphics. The parametric representation of curves is the most convenient for design specially in polynomial form. The Bezier curve is a parametric curve, a polynomial functions present it in certain parameter \( \theta \). In Computer aided Geometric Design (CAGD), a Bezier form is used to present the curve. The number of control points determine the degree of polynomial. The curve passes through its endpoints and it does not pass through the inner points. Each inner point attracts the curve towards itself and it causes change the direction of the curve. The polygon obtained when the control points are linked with straight lines are
called control polygon [1]. A rational Bezier curve is special case of non-uniform rational B-spline (NURBS). The rational control points are linked with straight lines are called control polygon. One of the important application of CAGD is construction free form of curve [2]. The curve is the important part for the engineers. They use some designing technique of CAGD to determine the shapes of designer. A rational Bezier curve have geometrically meaningful presentations so they are used it in CAD and CAGD [3]. The effect with changing the weight and moving the control points on the rational Bezier curve cause a global change in the shape of the curve, for that reason some authors developed methods by combination the shape parameters into the original basis functions, see [4] to [10]. These new curve have the same properties of Bezier curve.

The Arabic character is represent in this paper using Bezier-Ball Like curve. The Arabic script is written from right to left and it is generated using many segments which depend on the shape of letters. The two important things to generate Arabic script are the control points and number of segments. Many authors have been worked in design Arabic fonts. Sarfaz [11] introduced algorithm for automatic capture of Arabic fonts. In this work the rational Bezier-Ball curve is proposed. The beauty of Bezier-Ball Like curve can represent the Bezier curve, Ball curve, and Timmer curve at specific value of shape parameters. The shape of the curve can be modify by change the value of shape parameters and weight.

2 New Basis Functions

The cubic basis functions with two shape parameters that satisfies all properties of blending functions are define as a following:

\[ f_0 = (1 - \gamma \theta) (1 - \theta)^2 \]
\[ f_1 = (2 + \gamma) (1 - \theta)^2 \theta \]
\[ f_2 = (2 + \mu) (1 - \theta)^2 \theta^2 \]
\[ f_3 = (1 - \mu (1 - \theta)) \theta^2 \]

Where \( \theta \in [0, 1] \) and \( \gamma, \mu \in [-2, 1] \) so that the function will be positive. Figure (1) shows the basis functions with different value of \( \gamma, \mu \) in the range \([ -2, 1] \).
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For $\gamma = 1, \mu = -1$ (dotted lines), and for $\gamma = -1, \mu = 1$ (solid lines)

![Graph of Cubic Bezier-Ball Like curve]

Figure 1: Basis functions of Bezier-Ball Like curve

3 Basis Function with Specific Value of Shape Parameters

At specific value of $\gamma$ and $\mu$ will produced three cases of basis functions of curve.

- **Case 1**
  By setting $\gamma = \mu = 1$ will reduce the basis functions of Bezier curve as follows:

  
  \[
  f_0 = (1 - \theta)^3 \\
  f_1 = 3(1 - \theta)^2 \theta \\
  f_2 = 3(1 - \theta) \theta^2 \\
  f_3 = \theta^3
  \]  
  (3.1)

- **Case 2**
  By putting $\gamma = \mu = 0$ will get the basis function of Ball curve as in equations:

  \[
  f_0 = (1 - \theta)^2 \\
  f_1 = 2(1 - \theta)^2 \theta \\
  f_2 = 2(1 - \theta)\theta^2 \\
  f_3 = \theta^2
  \]  
  (3.2)

- **Case 3**
  By choosing the value of $\gamma = \mu = 2$ will reduce the basis functions to Timmer basis functions but the value of $\gamma, \mu$ not on the interval $[-2, 1]$ as:

  \[
  f_0 = (1 - 2\theta)(1 - \theta)^2 \\
  f_1 = 4(1 - \theta)^2 \theta
  \]  
  (3.3)
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\[ f_2 = 4(1 - \theta)\theta^2 \]

\[ f_3 = (2\theta - 1)\theta^2 \]

(a)

(b)
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Figure 2: The Specific Value of Shape Parameters (a) Bezier cubic basis functions for $\gamma=\mu=1$, (b) Ball cubic basis functions for $\gamma=\mu=0$, (c) Timmer cubic basis functions for $\gamma=\mu=2$.

4 Properties of Bezier-Ball like Curve Basis Functions

The basis functions (2.1) satisfies the following properties:

• **Positivity**

For $-2 \leq \gamma, \mu \leq 1$, the basis function (2.1) are non-negative on the interval $0 \leq \theta \leq 1$ but this property does not satisfy for value outside the domain of $\gamma, \mu$ see Figure 1.

• **Partition of unity**

The sum of Bezier-Ball like Basis Functions is one on the interval $0 \leq \theta \leq 1$.

$$\sum_{i=0}^{3} f_i(\theta) = (1 - \gamma \theta)(1 - \theta)^2 + (2 + \gamma)(1 - \theta)^2 \theta + (2 + \mu)(1 - \theta)^2 \theta^2 + (1 - \mu)(1 - \theta)^2 \theta^3 = 1$$

• **Monotonicity**

$f_0(\theta)$ is monotonically decreasing and $f_3(\theta)$ is monotonically increasing for given value of $\gamma$ and $\mu$. See Figure 3 for given value $\gamma = -0.5, \mu = 0.5$ $f_0$ is monotonically decreasing and $f_3$ is monotonically increasing.
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Figure 3: Monotonically of basis function

• Symmetry
Let \( f_i(\theta; \gamma, \mu) = f_{3-i}(1 - \theta; \mu, \gamma) \), \( i = 0, 1, 2, 3 \).

For \( i = 3 \)

\[ f_3(\theta; \gamma, \mu) = (1 - \mu(1 - \theta))\theta^2 = (1 - \gamma \theta)(1 - \theta)2 = f_0(1 - \theta; \mu, \gamma). \]

5 The Rational Cubic Bezier-Ball like Curve
The rational cubic Bezier-Ball Like curve with two shape parameters for given control points \((P_i, i = 0, 1, 2, 3)\) is defined as:

\[ r(\theta) = \frac{\sum_{i=0}^{3} w_i P_i f_i(\theta)}{\sum_{i=0}^{3} w_i f_i(\theta)} \quad (5.1) \]

Where \( \theta \in [0, 1] \), \( w_i \) are a weights in standard representation where \( w_0 = 1, w_3 = 1 \) and \( w_1, w_2 \) are positive.

6 Properties of Rational Cubic Bezier-Ball like Curve
The properties of rational cubic Bezier-Ball Like curve satisfies the following properties:

• End Point Properties
\[ r(0) = P_0, \quad r(1) = P_3 \]
\[ r'(0) = (2 + \gamma)(P_1 - P_0)w_1 \]
\[ r'(1) = (2 + \mu)(P_3 - P_2)w_2 \]
\[ r''(0) = 2((1 - \mu)P_3 + (2 + \mu)P_2w_2 - (2 + \gamma)P_1w_1(-\gamma + (2 + \gamma)w_1) + P_0(-1 + \mu - 2 + \mu)w_2 + (2 + \gamma)w_1(-\gamma + (2 + \gamma)w_1)) \]
\[ r''(1) = 2((1 - \gamma)P_0 + (2 + \gamma)P_1w_1 - (2 + \mu)P_2w_2(-\mu + (2 + \mu)w_2) + P_3(-1 \]
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\[ + \gamma - (2 + \gamma)w_1 + (2 + \mu)w_2 (-\mu + (2 + \mu)w_2) \]

- **Convex Hull Properties**
  For value of \(-2 \leq \gamma, \mu \leq 1\) the curve confined to the convex hull of its control points and it does not satisfy for values of \(\gamma, \mu\) outside the domain.

- **Symmetry**
  Let \(r (\theta; \gamma, \mu, P_i) = r (1 - \theta; \mu, \gamma, P_{3-i})\)
  That mean the control points \(P_i, P_{3-i}\) defined the same rational Bezier-Ball Like cubic curve with different parameterizations.

- **Geometric Invariance**
  The invariance of the shape of rational Bezier-Ball Like cubic curve are protected by the partition of unity property of Bezier-Ball Like basis functions under translation and rotation of its control points.

### 7 The Rational Cubic Bezier-Ball like Curve with Shape Control

The weights \(w_1, w_2\) and the two shape parameters \(\gamma, \mu\) control the shape of rational cubic Bezier Ball Like curve. Let the control points are given, taking \(w_1, w_2\) and \(\mu\) constants, the curve will become closer to \(P_0P_1\) such that \(\gamma\) increasing in the range \([-2, 1]\) see Figure 4(a). For constants \(w_1, w_2\) and \(\gamma\) the curve will moves towards \(P_2P_3\) where \(\mu\) increasing in the range \([-2, 1]\) as shown in Figure 4(b), and if \(\gamma, \mu\) change simultaneously with fixed \(w_1, w_2\) then the curve will increase toward the control polygon Figure 4(c). By the same manner for fixed values of \(\gamma, \mu\) with change the values of \(w_1\) and \(w_2\). For taking \(w_1\) constant with increase \(w_2\) from small value to big value the curve will adjacent to right side (moves toward \(P_2P_3\)). And fixed \(w_2\) with increase \(w_1\) from small to big the curve will become closer to left side (moves towards \(P_0P_1\)) see Figure 4(d).

### 8 Approximation of Rational Cubic Bezier-Ball like Curve

In this section, we will illustrate to approximate the rational cubic Bezier-Ball like curve by choosing different values of \(\gamma\) and \(\mu\). For values of \(\gamma, \mu\) equal to one will reduce cubic Bezier curve (red), also will produce cubic Ball curve if values of \(\gamma, \mu\) equal to zero (green) and for values of \(\gamma, \mu\) equal to two will get cubic Timmer curve (blue) see Figure 5.

### 9 Applications of Non-Rational Cubic Bezier-Ball like Curve

This section will discuss some applications for non-rational cubic Bezier-
Ball Like curve such as watermelon and font design using Matlab software.

Figure 4: The Shape Control of Rational Bezier-Ball Like curve (a) Change the value of $\gamma$, (b) Change the value of $\mu$, (c) Change the value of $\gamma$, $\mu$, (d) Change the value of $w_1$ and $w_2$.

Figure 5: Change the values of $\gamma$, $\mu$ produce different curves.
9.1 First Application (Arabic Font)

The way for drawing font require the control points for all segment that determine the shape of the letter. After that it is required to apply suitable programming language with non rational cubic Bezier-Ball Like curve for all segment to produce appropriate form of the letter. To adjust the shape of letter must change the inner points for all segments.

9.1.1 Numerical Example

Example1: In this section, we will show some examples of Arabic letters using Matlab software to implement non-rational cubic Bezier-Ball Like curve.

Design (Baa) letter via 21 segment of non-rational cubic Bezier-Ball Like curve by using Matlab see Figures 6.

Figure 6: Baa letter using non-rational cubic Bezier-Ball Like curve with control polygon

Example2: Design (Ain) letter via 14 segment of non-rational cubic Bezier-Ball Like curve by using Matlab see Figures 7.
Figure 7: Ain letter using non-rational cubic Bezier-Ball Like curve with control polygon

9.2 Second Application (Watermelon)

By using nine segment of non-rational cubic Bezier-Ball Like curve and different value of $\gamma$, $\mu$ for drawing watermelon see Figure 8.

Figure 8: Draw watermelon using non-rational cubic Bezier-Ball Like curve.

10 Results Discussion

This paper is organized as a follows. In section 2 the new basis functions
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with two shape parameters were established. In section 3 the basis functions with specific value of shape parameters were shown. In section 4 discussed the geometric properties of Bezier-Ball Like curve. Section 5, included the rational cubic Bezier-Ball Like curve and geometric properties were discussed in section 6. Section 7 provided a shape control of rational cubic Bezier-Ball Like curve and had shown that the two shape parameters were suitable instrument to control the shape of the curve and its application in font designing was given in section 9. The result of this study indicate that the important things to generate arabic script were the control points and the number of segments. In addition it was found that by generating the arabic font using non-rational cubic Ball-Bezier curve was appeared similar to the original font. In this study, we investigated the arabic font design apparently very close to the shape of the original font and similar to result that discovered by another authers.

11 Conclusion

As mentioned above, one of the more significant finding to emerge from this work is that the present study confirms previous finding and contributes additional evidence that suggests a new cubic basis functions which inherits all properties of Bezier curve. Since there is no difference in construct between Bezier-Ball Like and Bezier curve, it is not difficult to adopt proposed curve to a CAD that already use Bezier curve. The designed curve is used to generate Arabic alphabet that given more evidence of the claim. This work can be extended to prove that the rational cubic Ball-Bezier curve is more flexible than the rational quadratic Bezier-Like curve by generating arabic script for two ways to compare which one is better in terms of smoothness and by increasing number of segments.

References


